8.4

25) $\int_0^2 \frac{dx}{1-x^2} = \int_0^1 \frac{dx}{1-x^2} + \int_1^2 \frac{dx}{1-x^2}$

$\lim_{b\to 1^-} \int_0^b \frac{dx}{1-x^2} + \lim_{b\to 1^+} \int_b^2 \frac{dx}{1-x^2}$

$\lim_{b\to 1^-} \Big|_0^b \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + \lim_{b\to 1^+} \Big|_b^2 \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$

$\lim_{b\to 1^-}\left[\underbrace{\frac{1}{2}\ln\left|\frac{1+b}{1-b}\right|}_{\infty} - \underbrace{\frac{1}{2}\ln\frac{1+0}{1-0}}_{0}\right] + \lim_{b\to 1^+}\left[\underbrace{\frac{1}{2}\ln\left|\frac{1+2}{1-2}\right|}_{\frac{1}{2}\ln 3} - \underbrace{\frac{1}{2}\ln\left|\frac{1+b}{1-b}\right|}_{\infty}\right]$

$\boxed{\text{DIVERGES}}$

$\int \frac{1}{(1-x)(1+x)}$ $\quad \frac{A}{1-x} + \frac{B}{1+x} = \frac{1}{1-x^2}$

$A(1+x) + B(1-x) = 1$

$x=1 \quad 2A = 1 \implies A = \frac{1}{2}$

$x=-1 \quad 2B = 1 \implies B = \frac{1}{2}$

$u = 1-x$
$du = -dx$

$\int \frac{\frac{1}{2}}{1-x}dx + \int \frac{\frac{1}{2}}{1+x}dx$

$-\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x| \to \frac{1}{2}(\ln|1+x| - \ln|1-x|)$

$\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$

8.4

(17) $\int_1^\infty xe^{-2x}\,dx = \lim_{b\to\infty} \int_1^b xe^{-2x}\,dx$

$\lim_{b\to\infty} \Big|_1^b -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$

$\lim_{b\to\infty} \left[-\frac{1}{2}be^{-2b} - \frac{1}{4}e^{-2b}\right] - \left[-\frac{1}{2}(1)e^{-2} - \frac{1}{4}e^{-2}\right]$

$\frac{3}{4e^2}$ $\qquad \lim_{b\to\infty} \left[-\frac{1}{2}\underbrace{\frac{b}{e^{2b}}}_{0} - \underbrace{\frac{1}{4e^{2b}}}_{0}\right] - \left[-\frac{1}{2e^2} - \frac{1}{4e^2}\right]$

$\boxed{\frac{1}{2e^2} + \frac{1}{4e^2}}$

$u = x \qquad v = -\frac{1}{2}e^{-2x}$

$du = dx \qquad dv = e^{-2x}\,dx$

$\int u\,dv = uv - \int v\,du$

$-\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}\,dx$

$-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$

$\frac{1}{0^2}$

8.4

(35) $\int_0^{\ln 2} y^{-2} e^{1/y}\,dy = \lim_{a\to 0^+} \int_a^{\ln 2} y^{-2} e^{1/y}\,dy$

$\lim_{a\to 0^+} \Big|_a^{\ln 2} -e^{1/y}$

$\lim_{a\to 0^+} \left[-e^{1/\ln 2} + e^{1/a}\right]$

$e^{1/0}$

e^{∞}

$\boxed{\text{DIVERGES}}$

$\int y^{-2} e^{1/y}\,dy$ du

$u = \frac{1}{y} = y^{-1}$

$du = -y^{-2}\,dy$

$-\int e^u\,du$

$-e^u$

$-e^{1/y}$